# INFLUENCE OF THERMOGRAVITATIONAL FORCES ON TURBULENT TRANSPORT FOR VARIOUS ORIENTATIONS OF THE FLOW AND THE SURFACE AROUND WHICH THE FLOW IS OCCURRING

V. M. Eroshenko, L. N. Zaichik,
and V. A. Pershukov

UDC 532.517:536.24

On the basis of the second-moment balance equations, the influence of free convection on the intensity of turbulent transport in the boundary layer for various orientations of the flow and the surface around which the flow is occurring is studied.

An essential characteristic of turbulent flow in a gravitational field is the fact that the turbulent flow is generated by various mechanisms: via the operation of thermogravitational forces and by a shift in the mean velocity. The way in which the thermogravitational forces influence the intensity of turbulent transport is to a great extent determined by the orientation of the system in the mass force field. The influence of the gravitational force on the characteristics of turbulence in horizontal and vertical layers has been discussed in several papers (for example, [1-6]). An analysis of the effect of thermogravitational forces on the intensity of turbulent momentum and heat transport in the boundary layer for various orientations of the flow and the surface around which the flow is occurring is presented.

1. Let us choose a coordinate system as shown in Fig. 1, where the velocity vector of the average flow is directed along the x axis. In this case, the direction in which the gravitational force acts is characterized by the values of two angles: $\alpha$, the angle between the vector $\vec{g}$ and its projection on the plane of flow; and $\gamma$, the angle between the projection of $\vec{g}$ on the Oxz plane and the z axis. In much the same way as in [1-6], the analysis is carried out starting from the balance equations for the second moments of the fluctuations in velocity and temperature, which, in the Boussinesq approximation, are of the form

$$\frac{D\langle u_i'u_j'\rangle}{Dt}+\langle u_i'u_k'\rangle\frac{\partial U_j}{\partial x_k}+\langle u_j'u_k'\rangle\frac{\partial U_i}{\partial x_k}-\langle\frac{p'}{\rho}\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)\rangle+ 2\nu\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\rangle+\frac{\partial}{\partial x_k}\left(\langle u_i'u_j'u_k'\rangle-\nu\frac{\partial\langle u_i'u_j'\rangle}{\partial x_k}+\frac{\langle p'u_j'\rangle\delta_{ik}}{\rho}+ \frac{\langle p'u_i'\rangle}{\rho}\delta_{jk}\right)-\beta_\rho\left(g_i\langle u_j'\vartheta'\rangle+g_j\langle u_i'\vartheta'\rangle\right)=0, \quad (1)$$

$$\frac{D\langle u_i'\vartheta'\rangle}{Dt}+\langle u_i'u_k'\rangle\frac{\partial T}{\partial x_k}+\langle u_k'\vartheta'\rangle\frac{\partial U_i}{\partial x_k}-\langle\frac{p'}{\rho}\frac{\partial\vartheta'}{\partial x_k}\rangle+ (\nu+a)\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial\vartheta'}{\partial x_k}\rangle+\frac{\partial}{\partial x_k}\left(\langle u_i'u_k'\vartheta'\rangle-a\langle u_i'\frac{\partial\vartheta'}{\partial x_k}\rangle- \nu\langle\vartheta'\frac{\partial u_i'}{\partial x_k}\rangle+\frac{\langle p'\vartheta'\rangle}{\rho}\delta_{ik}\right)-\beta_\rho g_i\langle\vartheta'^2\rangle=0, \quad (2)$$

$$\frac{D}{Dt}\langle\vartheta'^2\rangle+2\langle u_k'\vartheta'\rangle\frac{\partial T}{\partial x_k}+2a\langle\frac{\partial\vartheta'}{\partial x_k}\frac{\partial\vartheta'}{\partial x_k}\rangle+\frac{\partial}{\partial x_k}\left(\langle u_k'\vartheta'^2\rangle-a\frac{\partial\langle\vartheta'^2\rangle}{\partial x_k}\right)=0, \quad (3)$$

where the $g_i$ are the components of the gravitational force vector: $g_x = g\cos\alpha\sin\gamma$, $g_y = g\sin\alpha$, and $g_z = g\cos\alpha\cos\gamma$.

G. M. Krzhizhanovskii Energy Institute, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 3, pp. 413-421, September, 1985. Original article submitted May 21, 1984.

0022-0841/85/4903-1051$09.50 

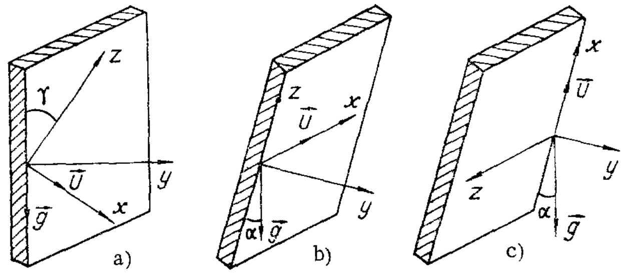


Fig. 1. Orientation of the system in the gravitational field.

We shall now analyze the influence of the lift forces on the mechanism of turbulent transport at high turbulent Reynolds numbers under the condition that the gradients of the averaged velocities and temperatures along the normal to the surface be much larger than the derivatives along the other directions. To describe the dissipative terms in Eqs. (1)-(3), as well as the correlations between the pressure fluctuations and the derivatives of the velocity and temperature, we shall use the simple approximate relations [7-9].

$$\nu \left\langle \frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k} \right\rangle = \frac{c}{3} \frac{E^{3/2}}{l} \delta_{ij}, \qquad a \left\langle \frac{\partial \vartheta'}{\partial x_k} \frac{\partial \vartheta'}{\partial x_k} \right\rangle = \frac{c_\vartheta}{l} E^{1/2} \langle \vartheta'^2 \rangle,$$

$$-\left\langle \frac{p'}{\rho} \left( \frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right) \right\rangle = \frac{k}{l} E^{1/2} \left( \langle u_i' u_j' \rangle - \frac{2}{3} E \delta_{ij} \right), \qquad -\left\langle \frac{p'}{\rho} \frac{\partial \vartheta'}{\partial x_i} \right\rangle = \frac{k_\vartheta}{l} E^{1/2} \langle u_i' \vartheta' \rangle.$$

There are no terms which take the influence of thermogravitational forces into account [1] because introducing them makes the analysis we are making more complicated and requires more constants to be determined (without changing the results obtained significantly).

If the distributions of the averaged velocity and temperature, as well as the scale of the turbulence are known, system (1)-(3) is a closed system of algebraic equations from which all of the pulsation moments may be determined:

$$\langle u_x' u_y' \rangle \frac{\partial U}{\partial y} + \frac{k}{2} \frac{E^{1/2}}{l} \left( \langle u_x'^2 \rangle - \frac{2}{3} E \right) + \frac{c}{3} \frac{E^{3/2}}{l} - g\beta_\rho \cos\alpha \sin\gamma \langle u_x' \vartheta' \rangle = 0, \tag{4}$$

$$\frac{k}{2} \frac{E^{1/2}}{l} \left( \langle u_y'^2 \rangle - \frac{2}{3} E \right) + \frac{c}{3} \frac{E^{3/2}}{l} - g\beta_\rho \sin\alpha \langle u_y' \vartheta' \rangle = 0, \tag{5}$$

$$\frac{k}{2} \frac{E^{1/2}}{l} \left( \langle u_z'^2 \rangle - \frac{2}{3} E \right) + \frac{c}{3} \frac{E^{3/2}}{l} - g\beta_\rho \cos\alpha \cos\gamma \langle u_z' \vartheta' \rangle = 0, \tag{6}$$

$$\langle u_y'^2 \rangle \frac{\partial U}{\partial y} + \frac{k}{l} E^{1/2} \langle u_x' u_y' \rangle - g\beta_\rho \left( \langle u_x' \vartheta' \rangle \sin\alpha + \langle u_y' \vartheta' \rangle \cos\alpha \sin\gamma \right) = 0, \tag{7}$$

$$\langle u_y' u_z' \rangle \frac{\partial U}{\partial y} + \frac{k E^{1/2}}{l} \langle u_x' u_z' \rangle - g\beta_\rho \left( \langle u_z' \vartheta' \rangle \cos\alpha \sin\gamma + \langle u_x' \vartheta' \rangle \cos\alpha \cos\gamma \right) = 0, \tag{8}$$

$$\frac{k E^{1/2}}{l} \langle u_y' u_z' \rangle - g\beta_\rho \left( \langle u_z' \vartheta' \rangle \sin\alpha + \langle u_y' \vartheta' \rangle \cos\alpha \cos\gamma \right) = 0, \tag{9}$$

$$\langle u_y'^2 \rangle \frac{\partial T}{\partial y} + k_\vartheta \frac{E^{1/2}}{l} \langle u_y' \vartheta' \rangle - g\beta_\rho \sin\alpha \langle \vartheta'^2 \rangle = 0, \tag{10}$$

$$\langle u_x' u_y' \rangle \frac{\partial T}{\partial y} + \langle u_y' \vartheta' \rangle \frac{\partial U}{\partial y} + \frac{k_\vartheta E^{1/2}}{l} \langle u_x' \vartheta' \rangle - g\beta_\rho \cos\alpha \sin\gamma \langle \vartheta'^2 \rangle = 0, \tag{11}$$

$$\langle u_z' u_y' \rangle \frac{\partial T}{\partial y} + k_\vartheta \frac{E^{1/2}}{l} \langle u_z' \vartheta' \rangle - g\beta_\rho \cos\alpha \cos\gamma \langle \vartheta'^2 \rangle = 0, \tag{12}$$

$$\langle u_y' \vartheta' \rangle \frac{\partial T}{\partial y} + c_\vartheta \frac{E^{1/2}}{l} \langle \vartheta'^2 \rangle = 0. \tag{13}$$

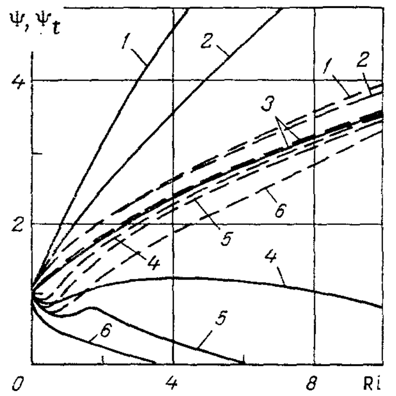


Fig. 2

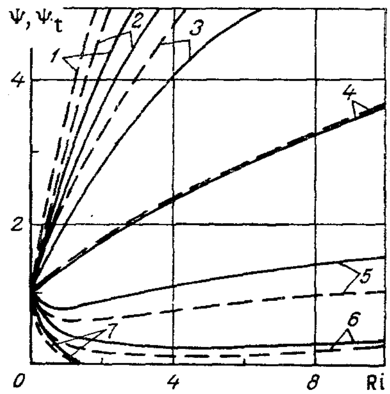


Fig. 3

Fig. 2. Influence of the angle $\gamma$ and the Richardson number on the intensity of turbulent momentum (solid lines) and heat (dashed lines) transport for flow along a vertical plate: 1) $\gamma = \pi/2$; 2) $\pi/6$; 3) 0, 4) $-\pi/6$; 5) $-\pi/3$; 6) $-\pi/2$.

Fig. 3. Influence of the angle of inclination of the surface $\alpha$ and the Richardson number on the intensity of turbulent momentum (solid lines) and heat (dashed lines) transport for transverse flow: 1) $\alpha = -\pi/2$; 2) $-\pi/3$; 3) $-\pi/6$; 4) 0, 5) $\pi/6$; 6) $\pi/3$; 7) $\pi/2$.

As a result, the solutions to this system may be given by the Prandtl or Kolmogorov equations for the tangential turbulent stress and transverse turbulent heat flow

$$\langle u'_x u'_y \rangle = -\frac{2(1-\bar{c})\left[1+\cos\alpha\sin\gamma R-\left(1-\dfrac{\bar{k}_\vartheta}{\bar{c}_\vartheta}\times(1-R\cos\alpha\sin\gamma)\right)\sin\alpha R^2/\mathrm{Ri}\right]\mathrm{Ri}}{3kk_\vartheta(1+\bar{k}_\vartheta\sin\alpha R^2/\mathrm{Ri})\left[1+\left(2+\dfrac{1}{\bar{c}_\vartheta}\right)\times\bar{k}_\vartheta\sin\alpha R^2/\mathrm{Ri}\right]R}\, l^2\left(\frac{\partial U}{\partial y}\right)^2 =$$
$$= -\Psi l^2\left(\frac{\partial U}{\partial y}\right)^2 = -\frac{Rk_\vartheta}{\mathrm{Ri}}\Psi l E^{1/2}\frac{\partial U}{\partial y} = -\nu_t\frac{\partial U}{\partial y}, \tag{14}$$

$$\langle u'_y\vartheta' \rangle = -\frac{2(1-\bar{c})\,\mathrm{Ri}}{3k_\vartheta^2[1+(2+1/\bar{c}_\vartheta)\bar{k}_\vartheta\sin\alpha R^2/\mathrm{Ri}]R}\, l^2\frac{\partial U}{\partial y}\frac{\partial T}{\partial y} =$$
$$= -\Psi_t\frac{l^2}{\bar{k}_\vartheta}\frac{\partial U}{\partial y}\frac{\partial T}{\partial y} = -\frac{Rk_\vartheta}{\mathrm{Ri}}\Psi_t l E^{1/2}\frac{\partial T}{\partial y} = -a_t\frac{\partial T}{\partial y}. \tag{15}$$

A relationship between the parameters R and Ri may be obtained from the balance equation for the turbulent energy, which may be obtained by combining equations (4)-(6):

$$\Psi\frac{\bar{k}_\vartheta}{\mathrm{Ri}}(1+R\cos\alpha\sin\gamma)+\Psi_t\left\{\left(1+\frac{\bar{k}_\vartheta}{\bar{c}_\vartheta}R\cos\alpha\sin\gamma\right)\cos\alpha\sin\gamma R/\mathrm{Ri}-\right.$$
$$\left.-\sin\alpha\right\}-\frac{\mathrm{Ri}^2\bar{c}}{R^3\bar{k}_\vartheta^2k^2}+\frac{R^2(1+1/\bar{c}_\vartheta)\bar{k}_\vartheta\cos\alpha\cos\gamma\Psi_t}{(1+\bar{k}_\vartheta\sin\alpha R^2/\mathrm{Ri})\mathrm{Ri}}=0. \tag{16}$$

The effects of the thermogravitational forces on the intensity of turbulent momentum and heat transport can be characterized using the way in which $\Psi$ and $\Psi_t$ depend on the Ri number. The values of the constants are chosen in accordance with experimental data for

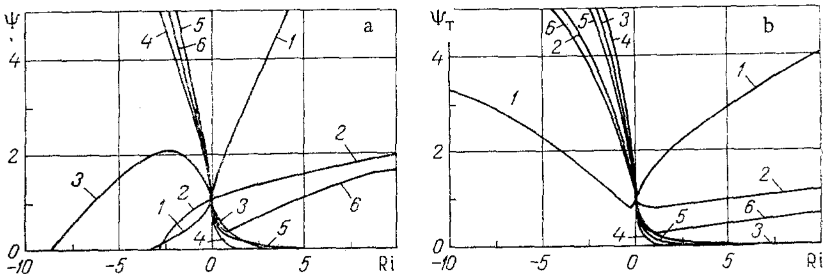


Fig. 4. Ψ(Ri) (a) and $\Psi_t$(Ri) (b) as a function of the angle at which the surface is inclined and the Richardson number for longitudinal flow: 1) 0, 2) π/6, 3) π/3, 4) π/2, 5) 2π/3, 6) 5π/6.

flows constrained by a wall: $\bar{c} = c/k = 0.125$, $\bar{c}_\vartheta = c_\vartheta/k = 0.2$, $\bar{k}_\vartheta = k_\vartheta/k = 0.9$; in addition, the normalization condition

$$\Psi(\mathrm{Ri}=0) = \Psi_t(\mathrm{Ri}=0) = \frac{1}{c^{1/2}}\left(\frac{2(1-\bar{c})}{3k}\right)^{3/2} = 1,$$

which corresponds to the condition that the turbulence scale in the absence of gravity be identical to the mixing length $\ell = \kappa y$ ($\kappa = 0.4$), i.e., $k = 1.12$, must be satisfied.

For small values of the Richardson number, the following expansions for Ψ and $\Psi_t$ can be obtained from Eqs. (14) and (15):

$$\Psi = 1 + \mathrm{Ri}\left\{\left(2+\frac{1}{2\bar{k}_\vartheta}\right)\sqrt{\frac{3\bar{c}}{2(1-\bar{c})}}\frac{\cos\alpha\sin\gamma}{\bar{k}_\vartheta} - \left(\frac{9\bar{c}(1+3\bar{k}_\vartheta)}{2(1-\bar{c})\bar{k}_\vartheta}+1\right)\frac{\sin\alpha}{2\bar{k}_\vartheta}\right\},$$
$$\Psi_t = 1 + \mathrm{Ri}\left\{\left(1+\frac{1}{2\bar{k}_\vartheta}\right)\sqrt{\frac{3\bar{c}}{2(1-\bar{c})}}\frac{\cos\alpha\sin\gamma}{\bar{k}_\vartheta} - \left(1+\frac{3\bar{c}(1+7\bar{k}_\vartheta+2\bar{k}_\vartheta/\bar{c}_\vartheta)}{2\bar{k}_\vartheta(1-\bar{c})}\right)\frac{\sin\alpha}{2\bar{k}_\vartheta}\right\}. \quad (17)$$

We shall now analyze the effects of the thermogravitational forces on the turbulent momentum and heat transport for the most characteristic positions of the system in the gravitational field: An arbitrarily oriented flow around a vertical heated surface (Fig. 1a), a transverse flow around a heated surface with an arbitrary spatial orientation (Fig. 1b), and a longitudinal flow around a surface with an arbitrary spatial orientation (Fig. 1c).

2. An arbitrarily oriented flow around a vertical surface (α = 0, γ variable). The functions Ψ(Ri) and $\Psi_t$(Ri) are shown for various values of the angle in Fig. 2. As follows from (17), a decrease in the intensity of turbulent momentum and heat transport is observed for γ < 0 (rising flow) while an increase is observed for γ > 0 (descending flow). The following asymptotic expressions may be obtained for large values of Ri:

$$\Psi = \frac{2(1-\bar{c})}{3k^2\bar{k}_\vartheta}(\chi^{-1/2}\mathrm{Ri}^{1/2}+\sin\gamma\,\mathrm{Ri}), \quad \Psi_t = \frac{2(1-\bar{c})}{3k^2\bar{k}_\vartheta}\chi^{-1/2}\mathrm{Ri}^{1/2}, \quad (18)$$

where

$$\chi = \sqrt{\frac{3\bar{c}}{2(1-\bar{c})\bar{k}_\vartheta^2(\sin^2\gamma+(1+1/\bar{c}_\vartheta)\cos\gamma+(1/\bar{c}_\vartheta)\sin^2\gamma)}}.$$

From the relations given above, it follows that growth in the intensity of momentum and heat transport for γ > 0. For γ < 0, $\Psi_t$ has an analogous asymptotic expression, and continues to grow, while the value of Ψ becomes negative. The values of the Richardson number for which Ψ ($\mathrm{Ri}_{cr}$) = 0 are given by the formula

$$\mathrm{Ri}_{cr} = -\frac{1}{\sin\gamma}\sqrt{\frac{2(1-\bar{c})}{3\bar{c}}\bar{k}_\vartheta\left(\frac{\cos\gamma}{\sin^2\gamma}\bar{k}_\vartheta(1+1/\bar{c}_\vartheta)+\left(\frac{\bar{k}_\vartheta}{\bar{c}_\vartheta}-1\right)\right)}.$$

The existence of regimes with negative values of Ψ is correlated with the presence of a negative coefficient of turbulent viscosity. In such regimes, the energy for supporting the averaged flow is transmitted from the fluctuational motion, which is, in turn, generated via the work of the thermogravitational forces. Such ("pathological" [10]) situations are frequently encountered when studying flows with additional sources of turbulent energy. Despite the existence of regimes with negative turbulent viscosity, the total generation of turbulent energy remains positive in all cases. It should be noted that in the limiting case of high Richardson numbers, a regime of thermogravitational turbulence generation in which the generation of turbulent energy occurs via shifts in the average velocity may be neglected. In these cases, the intensity of turbulent momentum and heat transport are given by (18). As follows from these equations, the asymptotical relations for $\Psi_t$ with $\gamma > 0$ and $\gamma < 0$ converge, which should lead to similar relations for the heat transfer Nu(Gr) for ascending and descending flows. In this case, the expression for the turbulent heat flow in the boundary layer is of the form

$$q_t = \frac{2(1-\bar{c})}{3k^2\bar{k}_\vartheta^2}\chi^{-1/2}c_p\rho(g\beta_\rho)^{1/2}l^2\left|\frac{\partial T}{\partial y}\right|^{3/2}. \tag{19}$$

From Eqs. (19) for the temperature profile in the turbulent boundary layer (for which we may set $q_t = q_w$ for both ascending and descending flows on a vertical surface), the "1/3 law"

$$T_+ = T_{+0} - A_1\xi^{-1/3} \tag{20}$$

holds for the region where free convection dominates. Such temperature distributions were obtained for descending ($\gamma = \pi/2$) and ascending ($\gamma = -\pi/2$) flows in [11-13]. Note that temperature distribution (20) yields a function of the form Nu ~ $\mathrm{Gr}^{1/4}$ for the heat transfer as a function of the Grashof number (this relation is in agreement with the well-known equations for calculating the heat transfer in the free-convection regime on vertical surfaces.

3. Transverse flow around an arbitrarily oriented surface ($\gamma = 0$, $\alpha$ variable). It follows from the asymptotic expansions for $\Psi$ and $\Psi_t$ at small Richardson numbers (17) that the intensity of turbulent momentum and heat transport decreases for $\alpha > 0$ (stable stratification), and, likewise, it increases for $\alpha < 0$ (unstable stratification). The following relations hold for Ri $\to \infty$:

$$\Psi = \frac{2(1-\bar{c})[1-(1-\bar{k}_\vartheta/\bar{c}_\vartheta)\chi\sin\alpha]}{3k^2\bar{k}_\vartheta(1+\bar{k}_\vartheta\sin\alpha\chi)[1+(2+1/\bar{c}_\vartheta)\sin\alpha\bar{k}_\vartheta\chi]\chi^{1/2}}\mathrm{Ri}^{1/2}, \tag{21}$$

$$\Psi_t = \frac{2(1-\bar{c})}{3k^2\bar{k}_\vartheta[1+(2+1/\bar{c}_\vartheta)\sin\alpha\bar{k}_\vartheta\chi]\chi^{1/2}}\mathrm{Ri}^{1/2},$$

where

$$\chi = \left\{-\left(1-\frac{3\bar{c}\left(3+\frac{1}{\bar{c}_\vartheta}\right)}{2(1-\bar{c})}\right)\sin\alpha + \left[\left(1-\frac{3\bar{c}\left(3+\frac{1}{\bar{c}_\vartheta}\right)}{2(1-\bar{c})}\right)^2\sin^2\alpha + \frac{6\bar{c}}{(1-\bar{c})\bar{k}_\vartheta}\left[1+\frac{\bar{k}_\vartheta\cos\alpha}{\bar{c}_\vartheta}-\sin^2\alpha\left(1-\frac{3\bar{c}\left(2+\frac{1}{\bar{c}_\vartheta}\right)}{2(1-\bar{c})}\right)\right]\right]^{1/2}\right\}\times$$
$$\times\left[2\left(1+\frac{\bar{k}_\vartheta\cos\alpha}{\bar{c}_\vartheta}-\sin^2\alpha\left(1-\frac{3\bar{c}\left(2+\frac{1}{\bar{c}_\vartheta}\right)}{2(1-\bar{c})}\right)\right)\right]^{-1}. \tag{22}$$

It follows from (21) that an increase in $\Psi$ and $\Psi_t$ is observed for both $\alpha > 0$ and $\alpha < 0$, except for the case $\alpha = \pi/2$. This behavior of the functions $\Psi$ and $\Psi_t$ is supported by the calculations shown in Fig. 3. In the case $\alpha = \pi/2$, which corresponds to flow on a horizontal surface with stable stratification, there is a critical Richardson number $\mathrm{Ri}_{cr}$ at which complete damping of the turbulence occurs. The existence of $\mathrm{Ri}_{cr}$ is supported by the experimental data given in [1, 14].

Note that as the Richardson number approaches the critical value, the intensity of turbulent heat transport is damped out more rapidly than the intensity of turbulent momentum transport, which results in a sharp increase in the turbulent Prandtl number $\mathrm{Pr}_t = \nu_t/a_t$. In the remaining cases with $\alpha \neq \pi/2$, as Ri $\to \infty$, the turbulent Prandtl number approaches a constant given by the following equation

$$\mathrm{Pr}_{t\infty} = \frac{1-\left(1-\frac{\bar{k}_\vartheta}{\bar{c}_\vartheta}\right)\chi\sin\alpha}{1+\bar{k}_\vartheta\sin\alpha\chi}\bar{k}_\vartheta,$$

where $\chi$ is defined as in Eq. (22).

From an analysis of the turbulent energy balance equations (16), it follows that a thermogravitational turbulence creation regime comes into existence at all values of the angle $\alpha \neq \pi/2$. For these regimes, we obtain "1/3 laws" for the mean temperature and velocity distributions of the form (20) in the boundary layer [2, 14].

4. Transverse flow around an arbitrarily oriented surface ($\gamma = \pi/2$, $\alpha$ variable). From an analysis of the equations for the intensity of turbulent momentum and heat transport at small values of the Richardson number (17), it follows that the action of free convection can lead to either an increase or a decrease in the intensity of turbulent transport, depending on the orientation of the motion with respect to the gravitational force and the sense of heat flow (heating or cooling).

The functions $\Psi(\mathrm{Ri})$ and $\Psi_t(\mathrm{Ri})$ are shown in Figs. 4a and b. For $\mathrm{Ri} > 0$, a decrease in the intensity of turbulent transport relative to the case discussed above (flow on a vertical surface ($\alpha = 0$)) is observed as the angle increases. In the region $\pi/4 \leqslant \alpha \leqslant 3\pi/4$, a monotonic decrease in the intensity of turbulent transport, while for $3\pi/4 \leqslant \alpha < \pi$, the values of $\Psi$ and $\Psi_t$ initially decrease as the number Ri increases, and then begin to increase.

The flow for $\mathrm{Ri} < 0$ for $0 < \alpha < \pi/2$ is characterized by the presence of negative viscosity. For $\alpha > \pi/2$, the flow becomes turbulent, which leads to a large increase in $\Psi$ and $\Psi_t$ with increasing Richardson number. It should be noted that for all $\alpha > 0$ and $\mathrm{Ri} < 0$, a thermogravitational turbulence creation regime, which is most prominent for angles $0 \leq \alpha < \pi/2$, comes into existence.

Thus, the case of longitudinal flow around an inclined surface includes all of the effects (the existence of $\mathrm{Ri}_{cr}$, the negative turbulent viscosity coefficient, thermogravitational turbulence generation) obtained when discussing the other special cases.

5. In conclusion, we note that for flows in the thermogravitational turbulence generation regime which take place on vertical and inclined heated surfaces, for upward flow on a vertical surface and one inclined at an angle $\alpha < \pi/3$ (and for flow on a horizontal surface with unstable stratification) the heat transport as a function of the Richardson number is given by the formula $\langle u'_y\vartheta'\rangle \sim \mathrm{Ri}^{1/2}$, which corresponds to a relationship of the form $q_t \sim \rho c_p(g\beta_\rho)^{1/2}\ell^2|\partial T/\partial y|^{3/2}$ and leads to a "1/3 law" for the temperature distribution in the averaged flow, and, accordingly, to a relationship of the form $\mathrm{Nu} \sim \mathrm{Gr}^{1/4}$ between the heat transfer and the free convection parameter.

It should be noted that the conclusion about the existence of negative turbulent viscosity was obtained under the condition that the system under discussion is in an equilibrium state, i.e., that it is possible to neglect the influence of turbulent diffusion and convection in the equations for the second moments. Taking diffusion and convection into account when solving the complete equations of turbulent transport may lead to some corrections to the results obtained above.

## NOTATION

x, y, and z, Cartesian coordinates; $U_i$ and T, averaged values of the velocity and temperature; $u_i'$, $\vartheta'$, $p'$, fluctuations in velocity, temperature, and pressure; $\nu_t$ and $a_t$, coefficients of turbulent viscosity and thermal conductivity; $\beta_\rho$, thermal expansion coefficients; E, fluctuation energy; $\delta$, thickness of the turbulent layer; $\ell$, turbulence scale; $L - c_p\rho U_*^3/\beta_\rho g q_w$, Monin–Obukhov scale; $U_* = \sqrt{\tau_w/\rho}$, dynamical velocity; $c_\vartheta$, c, $k_\vartheta$, and k, constants; $T_+ = (T - T_w)c_p\rho U_*/q_w$, temperature in universal coordinates, $\xi = y/L$, universal coordinate; $\mathrm{Ri} = g\beta_\rho\,\partial T/\partial y\,/\,(\partial U/\partial y)^2$, Richardson number; $\mathrm{Nu} = q_w\delta/(\lambda\Delta T)$, Nusselt number; $\mathrm{Gr} = g\beta_\rho q_w\delta^4/(\lambda\nu^2)$, Grashof number; and $R = g\beta_\rho l(\partial T/\partial y)\,/\,(\bar{k}_\vartheta E^{1/2}\times\partial U/\partial y)$.

## LITERATURE CITED

1. B. E. Launder, "Heat and mass transfer," in: Turbulence, P. Bradshaw (ed.), Springer-Verlag, Berlin (1976), pp. 231-287.

2. E. S. Monin, "The temperature-inhomogeneous boundary layer of the atmosphere," Fiz. Atmos. Okeana, 1, No. 5, 490-497 (1965).
3. K. E. Dzhaugashtin, "Critical flow regime in a stratified medium," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, 156-159 (1979).
4. V. N. Popov, "Influence of free convection on turbulent transport for flow of a liquid in a flat channel," Teplofiz. Vys. Temp., 21, No. 2, 281-291 (1983).
5. A. F. Polyakov, "Boundaries and the nature of the influence of thermogravitational forces on turbulent flow and heat exchange in vertical pipes," Teplofiz. Vys. Temp., 11, No. 1, 106-116 (1973).
6. V. N. Popov, "Influence of free convection on turbulent transport for flow of a liquid in a vertical channel," Teplofiz. Vys. Temp., 21, No. 3, 515-521 (1983).
7. A. S. Monin, "Symmetry properties of turbulence in the atmospheric boundary layer," Fiz. Atmos. Okeana, 1, No. 1, 45-55 (1965).
8. B. A. Kolovandin and V. E. Aerov, "Turbulent heat and mass transport in flows with a shift," in: Heat and Mass Transfer [in Russian], Vol. II, Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk (1969), pp. 66-87.
9. J. C. Rotta, "Statistische Teorie nichtmogener Turbulenz," Z. Phys., 129, No. 5, 547-572 (1951).
10. J. Mathieu and D. Jeandel, "Pathological cases in turbulent flows and the spectral approach," in: Prediction Methods for Turbulent Flows, Hemisphere Publ. Co., Washington, DC (1980).
11. A. F. Polyakov, "Flow and heat exchange in the thermogravitational generation regime," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 86-94 (1977).
12. W. K. George and S. P. Capp, "A theory for natural convection turbulent boundary layers next to heated vertical surfaces," Int. J. Heat Mass Transfer, 22, No. 6, 813-826 (1979).
13. Z. H. Queshi and B. Gebhart, "Transition and transport in a buoyancy-driven flow in water adjacent to a vertical uniform flux surface," Int. J. Heat Mass Transfer, 21, No. 12, 1467-1479 (1978).
14. B. S. Petukhov, A. F. Polyakov, and Yu. V. Tsypulev, "Turbulent momentum and heat transport in temperature-inhomogeneous stratified flow," in: Heat and Mass Transfer VI, Part I, Vol. 1 [in Russian], Minsk (1980), pp. 160-172.

# COUPLED SIMULTANEOUS HEAT AND MASS TRANSFER IN MULTICOMPONENT TWO-PHASE MIXTURES

L. P. Kholpanov, E. Ya. Kenig, and V. A. Malyusov

UDC 536.423.4:532.522.2

A method is proposed for calculating the parameters of simultaneous heat and mass transfer in a multicomponent two-phase gas–liquid system, this method being based on solving the system of differential equations of convective heat transfer and convective diffusion.

An important item in research concerning heat- and mass-transfer processes is development of a theory for simultaneous heat and mass transfer in multicomponent two-phase mixtures. Particular attention is paid to solution of this problem as a coupled one.

A method of solving such problems will be outlined here on the example of heat and mass transfer in a multicomponent two-phase gas–liquid system which flows through a vertical channel in the descending parallel-flow mode.

Let the x axis run along a channel wall and the y axis run perpendiclar to it. The thermal diffusivity of each component and the coefficients of multicomponent diffusion are

Institute of New Problems in Chemistry, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 3, pp. 421-428, September, 1985. Original article submitted August 6, 1984.

  